Undoing a quantum measurement

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In general, a quantum measurement yields an undetermined answer and alters the system to be consistent with the measurement result. This process maps multiple initial states into a single state and thus cannot be reversed. This has important implications in quantum information processing, where errors can be interpreted as measurements. Therefore, it seems that it is impossible to correct errors in a quantum information processor, but protocols exist that are capable of eliminating them if they affect only part of the system. In this work we present the deterministic reversal of a fully projective measurement on a single particle, enabled by a quantum error-correction protocol that distributes the information over three particles.

Measurements on a quantum system irreversibly project the system onto a measurement eigenstate regardless of the state of the system. Copying an unknown quantum state is thus impossible because learning about a state without destroying it is prohibited by the no-cloning theorem[1]. At first, this seems to be a roadblock for correcting errors in quantum information processors. However, the quantum information can be encoded redundantly in multiple particles and subsequently used by quantum error correction (QEC) techniques [2–7]. When one interprets errors as measurements, it becomes clear that such protocols are able to reverse a partial measurement on the system. In experimental realizations of error correction procedures, the effect of the measurement is implicitly reversed but its outcome remains unknown. Previous realizations of measurement reversal with known outcomes have been performed in the context of weak measurements where the measurement and its reversal are probabilistic processes[8–11]. We will show that it is possible to deterministically reverse measurements on a single particle.

We consider a system of three two-level atoms where each can be described as a qubit with the basis states $|0\rangle, |1\rangle$. An arbitrary pure single-qubit quantum state is given by $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$ and $\alpha, \beta \in \mathbb{C}$. In the used error-correction protocol, the information of a single (system) qubit is distributed over three qubits by storing the information redundantly in the state $\alpha|000\rangle + \beta|111\rangle$. This encoding is able to correct a single bit-flip by performing a majority vote and is known as the repetition code [12].

A measurement in the computational basis states $|0\rangle, |1\rangle$ causes a projection onto the σ_z axis of the Bloch sphere and can be interpreted as an incoherent phase flip. Thus, any protocol correcting against phase-flips is sufficient to reverse measurements in the computational basis. The repetition code can be modified to protect against such phase-flip errors by a simple basis change from $|0\rangle, |1\rangle$ to $|\pm\rangle = 1/\sqrt{2}(|0\rangle \pm |1\rangle)$. After this basis change each individual qubit is in an equal superposition of $|0\rangle$ and $|1\rangle$ and therefore it is impossible to gain any information about the encoded quantum information by measuring a single qubit along σ_z. Because the repetition code relies on a majority vote on the three-qubit register the measurement can be only perfectly corrected for if it acts on a single qubit as outlined in the schematic circuit shown in Fig. 1(a).

This process protects the information on the system qubit, leaving it in the same state as prior to the encoding. A complete reversal of the measurement brings the register back to the state it had immediately before the measurement. Therefore one needs to re-encode the register into the protected state. This is not directly possible because the ancilla qubits carry information about the measurement outcome. Therefore the auxiliary qubits have to be re-initialized prior to re-encoding as outlined in Fig. 1(a).

The experiment is realized in a linear chain of $^{40}\mathrm{Ca}^+$ ions confined in a macroscopic linear Paul trap[13]. Each ion encodes a qubit in the $4S_{1/2}(m = -1/2) = |1\rangle$ and the metastable $3D_{5/2}(m = -1/2) = |0\rangle$ state. Coherent manipulations of the qubit state are performed by exactly timed laser pulses in resonance with the energy difference between the two levels. A typical experimental sequence consists of (i) initialization of the quantum register, (ii) coherent state manipulation, and (iii) measurement of the register. Initializing the register consists of preparing the electronic state of the ions in a well defined state and cooling the common motional mode of the ions close to the ground state. In our experiment, any coherent operation can be implemented with a universal set of gates consisting of collective spin flips, phase shifts of individual qubits and collective entangling operations [14, 15].

The qubit can be measured in the computational basis by performing electron shelving on the short-lived $S_{1/2} \leftrightarrow P_{1/2}$ transition as sketched in Fig. 2(a). Here, projection onto the state $|1\rangle$ enables a cycling transition and scatters many photons if the detection light is applied, whereas after projection onto $|0\rangle$ no population

is resonant with the laser light at 397 nm. The outcomes can be distinguished by shining in the laser light long enough to detect multiple photons with a photomultiplier tube after projecting into $|1\rangle$. The absence of photons is then interpreted as outcome $|0\rangle$. Although the projection is already performed after scattering a single photon, it is necessary to detect multiple photons for faithful discrimination.

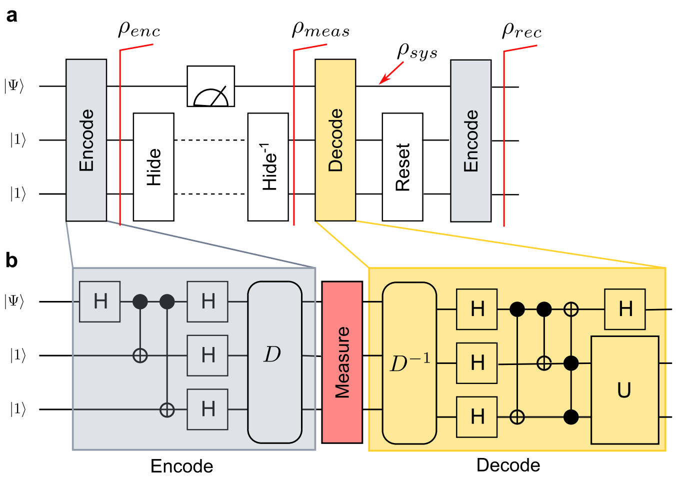

FIG. 1. (a) Schematic circuit of undoing a quantum measurement. ρ_{enc} is the encoded state of the register, ρ_{meas} is the state after the measurement, ρ_{sys} is the corrected state of the system qubit after the QEC cycle and ρ_{rec} is the state of the register after the full correction. (b) Circuit representation of the error correction algorithm. D is a unitary operation that commutes with phase flips. U is an arbitrary unitary operation. These operations do not affect the error correction functionality.

For the reversal scheme as shown in Fig. 1(a) only a single ion of the register is measured. This is realized by protecting the other two ions from the detection light by transferring the population from $|1\rangle$ in the $m = -5/2$ Zeeman substate of the $D_{5/2}$ level with the procedure outlined in Fig. 2(a) [16]. Then, a projective measurement does not affect the electronic state of the hidden ions which are the remaining carriers of the information. The uncertainty of the measurement on the remaining ion depends on how many photons are detected if the state was projected into $|1\rangle$. Given that the number of detected photons follow a Poissonian distribution, the detection uncertainty can be easily calculated via the cumulative distribution function of the Poisson distribution and the measurement durations as shown in columns one to three in table I.

The quality of subsequent coherent operations is significantly lowered by the recoil of the scattered photons heating the motional state of the quantum register. Therefore, recooling the ion-string close to the ground state is required without disturbing the quantum information in the non-measured qubits. In ion-traps this can

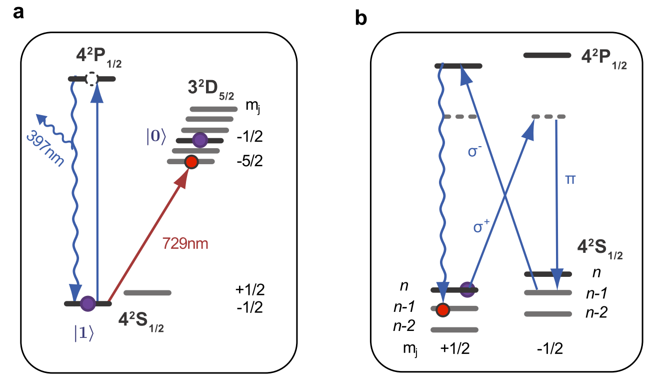

FIG. 2. (a) Schematic of the measurement process on the $S_{1/2} \leftrightarrow P_{1/2}$ transition. The auxiliary qubits are hidden from the measurement by transferring the population to the $m = -5/2$ substate of the $D_{5/2}$ level. (b) Schematic of the Raman recooling procedure. This scheme utilizes two 1.5GHz detuned Raman beams that remove one phonon upon transition from the Zeeman substates $m = -1/2$ to $m = +1/2$ and an additional resonant beam that is used to optically pump from $m = +1/2$ to $m = -1/2$ via the $P_{1/2}$ state.

be achieved with sympathetic cooling using a second ion species. As trapping and cooling two different ion species requires major experimental effort, we employ a recooling technique that can be used with a single trapped species. We perform a Raman cooling scheme as shown in Fig. 2(b) while the ancilla qubits are still protected.

Encoding and decoding of the register as shown in Fig. 1(b) are implemented in our setup as described in Ref.[17]. The encoding is realized with a single entangling operation and the decoding is performed using a numerically optimized decomposition into available operations [14]. In order to facilitate the optimization procedure, the QEC algorithm is slightly modified without affecting its functionality by two additional unitary operations D and U as shown in Fig. 1(b). The actual implementation can be benchmarked with the aid of quantum state and process tomography [12, 16]. We use a maximum likelihood algorithm to reconstruct the density matrix and perform a non-parametric bootstrap for statistical error analysis [18]. Because the error correction protocol acts as a single qubit quantum channel, it can be characterized by a quantum process tomography on the system qubit (indicated as ρ_{sys} in Fig 1(a)). This process is characterized by the process matrix χ_{exp} and its performance compared to the ideal process χ_{id} is given by the process fidelity $F^{proc} = \mathrm{Tr}(\chi_{id} \cdot \chi_{exp})$. The process fidelity of a single error correction step without measurement and recooling was measured to be $F = 93(2)\%$. The process including the measurement can be analyzed by either ignoring the measurement outcome or by investigating the process depending on the outcome as presented in Table I.

The overall performance of the reversal process is de-

termined by the quality of the operations and the loss of coherence during the measurement and the recooling process. As the quality of the operations is affected by the motional state of the ion-string after recooling, there is a trade-off between their fidelity and the loss of coherence during measurement and recooling. It should be noted that the measurement affects the motion only if it is projected into the $|1\rangle$ state whereas the loss of coherence affects both possible projections. The performance of the algorithm for different measurement and recooling parameters is shown in Table I. A detection error of less than 0.5% is achieved with a measurement time $\tau_{meas} = 200\mu s$ and a recooling time of $\tau_{recool} = 800\mu s$ leading to a mean process fidelity of $F = 84(1)\%$ which exceeds the bound for any classical channel of $F = 50\%$. We analyzed the measurement outcome for $\tau_{meas} = 200\mu s$ and a measurement threshold of three photon counts to show that no information about the encoded quantum information can be gained by measuring a single qubit. The measurement was performed for the initial basis states $|0\rangle$, $|0\rangle+|1\rangle$, $|0\rangle+i|1\rangle$, $|1\rangle$ and results in probabilities to find the outcome in state $|0\rangle$ of $48(1)\%, 50(1)\%, 50(1)\%, 50(1)\%$. This shows that indeed no information about the initial quantum state can be inferred by measuring a single qubit.

The presented procedure is able to protect the quantum information on the system qubit in the presence of a quantum measurement. In order to perform the full measurement reversal, the ancilla qubits have to be reset before applying the same encoding as demonstrated in Ref [17].

As this technique recovers the state of the entire register, the measurement reversal can be directly benchmarked by comparing the state before the measurement and after the reconstruction. A quantum state can be analyzed using quantum state tomography and evaluating the fidelity between two states ρ_1, ρ_2 with the Uhlmann fidelity[19] $F^{rho}(\rho_1, \rho_2) = (\mathrm{Tr}\ \sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}})^2$. The state ρ_{enc} after encoding shows a fidelity with the ideal state of $F(\rho_{id}, \rho_{enc}) = 94(1)\%$. In order to demonstrate the effect of the measurement the states ρ_{meas} after measuring and recooling, and ρ_{rec} after the reconstruction are analyzed with respect to the state ρ_{enc}. The measured density matrices for these states are shown in Fig. (3). The overlap of the state after the measurement ρ_{meas} with the state ρ_{enc} is $F(\rho_{enc}, \rho_{meas}) = 50(2)\%$ as expected from pure dephasing which shows that the measurement acts as dephasing when the outcome is ignored. In contrast, Fig. 3 illustrates the evolution of the states with known outcome. The reconstructed state ρ_{rec} after correction, reset and re-encoding is measured to have an overlap of $F(\rho_{enc}, \rho_{rec}) = 84(1)\%$ which indicates that the measurement was successfully reversed. The quality of the measurement reversal depends again on the number of scattered photons during the measurement and the recooling time and the optimum is also $\tau_{detect} = 200\mu s$. Fidelities depending on the outcome and for various measurement durations are displayed in table I.

In conclusion we have demonstrated the full reversal of a strong quantum measurement on a single qubit. We further presented an in-sequence recooling technique that can serve as an alternative to sympathetic two-species cooling. This may simplify the architecture for a future large-scale ion-trap quantum information processor.

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τ_{Raman}	τ_{meas}	Detection error	$\langle n_{phonon}\rangle$	F^{proc}_{mean}	$F^{proc}_{\|1\rangle}$	$F^{proc}_{\|0\rangle}$	F^{rho}_{mean}	$F^{rho}_{\|1\rangle}$	$F^{rho}_{\|0\rangle}$
800	100	4 %	0.17	86(3)	82(3)	90(2)	89(1)	87(1)	91(2)
800	200	< 0.5 %	0.24	85(2)	87(3)	90(3)	84(1)	82(1)	85(2)
800	300	< 0.5 %	0.41	81(3)	78(2)	87(3)	84(1)	80(1)	87(2)
800	400	< 0.5 %	0.50	78(3)	71(5)	85(4)	82(1)	76(2)	90(2)

TABLE I. Columns 1 to 3: Raman recooling and measurement duration in μs with corresponding detection error. Column 4: Measured mean phonon number $\langle n\rangle$ after measurement and recooling. Columns 5 to 7: Measured process fidelities on the system qubit without re-encoding F^{proc} in(%) and columns 8 to 10: Overlap of the quantum state after the full reconstruction with the state prior to the measurement F^{rho} in (%). Lower indices F_{mean} indicate a mean fidelity while ignoring the measurement outcome. $F_{|0\rangle}$ and $F_{|1\rangle}$ indicate fidelities if the measurement outcome was $|0\rangle$ ($|D\rangle$) and $|1\rangle$ ($|S\rangle$). Errors correspond to one standard deviation.

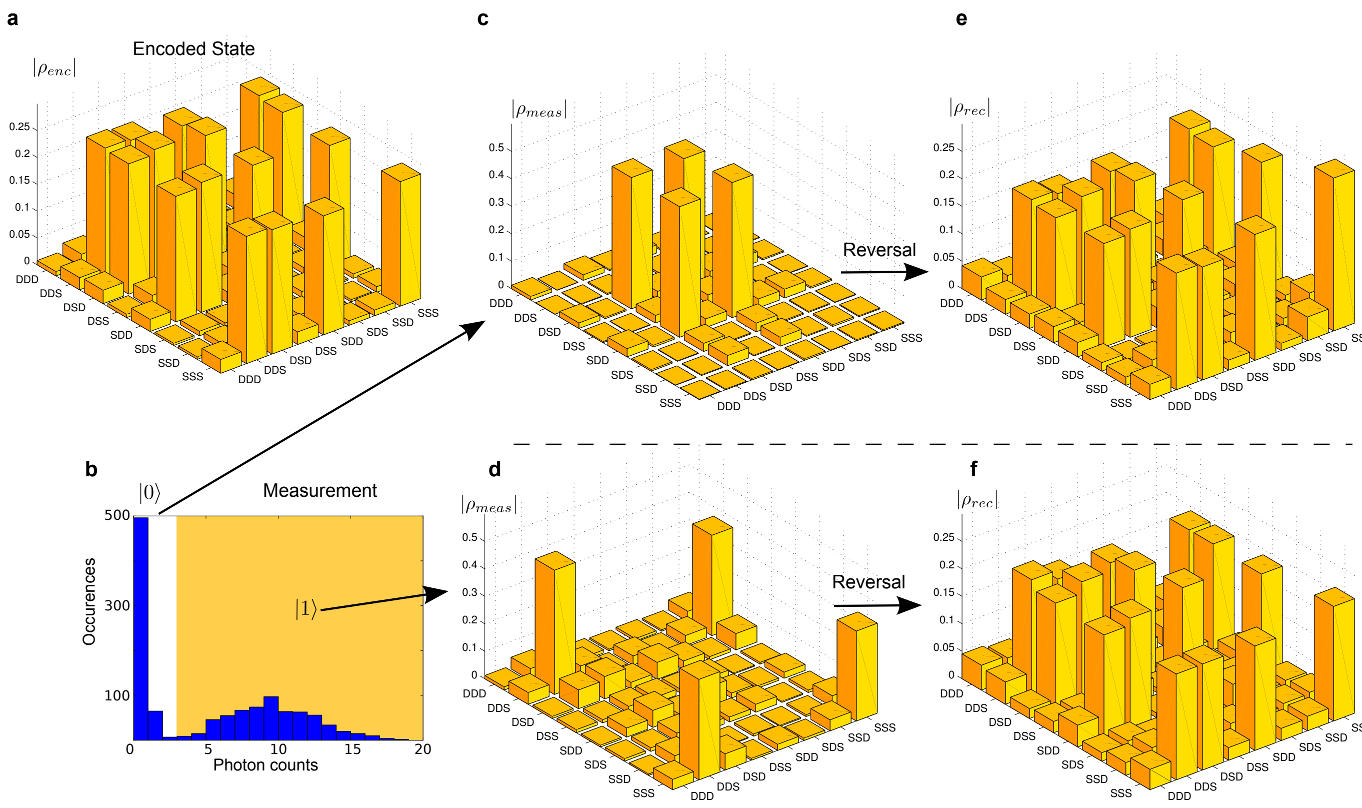

FIG. 3. (a) Absolute value of the reconstructed three qubit density matrices before the measurement ρ_{enc}. (b) Histogram of the measured photon counts for a measurement time of $200\mu s$. Absolute value of three qubit density matrices after the measurement ρ_{meas} for outcome (c) $|0\rangle$ and (d) $|1\rangle$. Density matrices after the measurement reversal ρ_{rec} for outcome (e) $|0\rangle$ and (f) $|1\rangle$.

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